M=A–T+H without tears!

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TEACHER PLUS ACTIVITIES

Teacher Plus Worksheet

Math without tears!

"If mathematics is not fun, something must be terribly wrong."

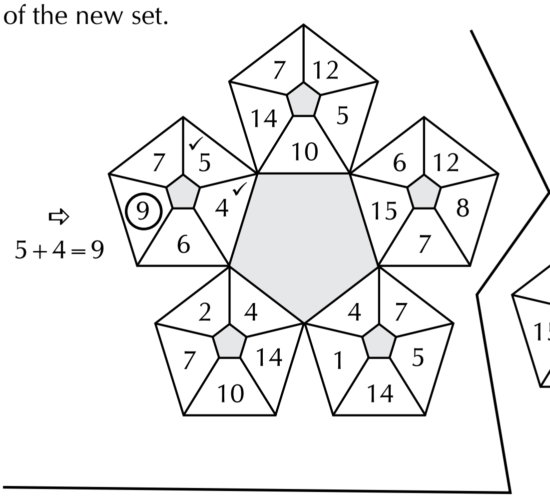
Anyone who has taught mathematics appreciates the need for practice in learning mathematics and achieving the mastery level. Practice helps in connecting concepts, application, strategy and critical thinking and is an important step in developing essential mathematical skills. However, even a cursory glance at the typical textbooks and workbooks in use, indicates the boring and monotonous nature of the practice work our children generally get.

The worksheets (rather funsheets!) in this set are a humble attempt to bring back the pleasure of working with mathematical ideas at the school level. Some of the ideas have been around for long but others are floating in the vast cyberspace and have been liberally included if appropriate. It is hoped that teachers teaching and children learning mathematics find this experience deeply engaging, insightful, relevant and effective.

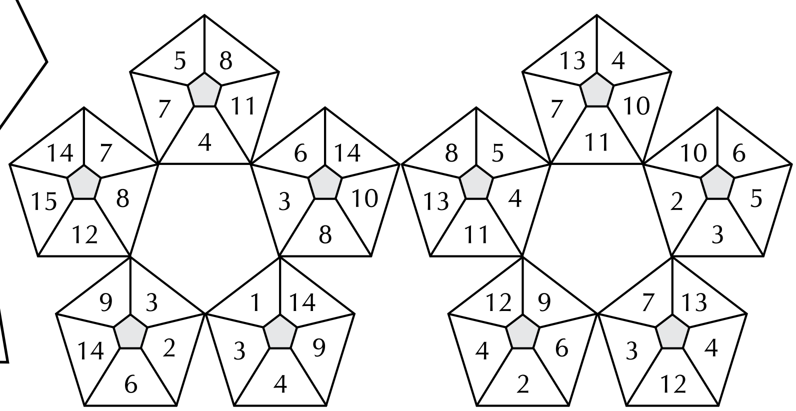
1. Kakooma

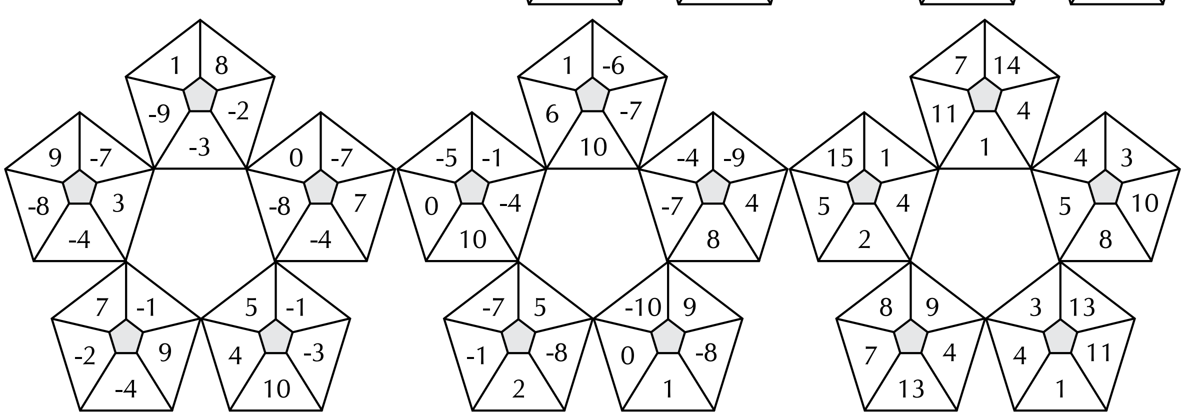
What is Kakooma? It is a simple mathematical activity that has gained enormous popularity in a short span of time. Teachers and kids like the Japanese game Kakooma – interestingly for entirely different reasons. One gets hooked to Kakooma easily as it is one of those "not-too-easy-or-difficult" ones.

Description: In this figure there are regular pentagons with five numbers in them. Here, find which number is the sum of TWO other numbers in that very pentagon. Mark it. Do it four more times and in that process, you will identify five numbers. Next, like a 'puzzle-in-a-puzzle', identify the number that is the sum of two other numbers of the new set.



Can you try these?





2. Casting out nines

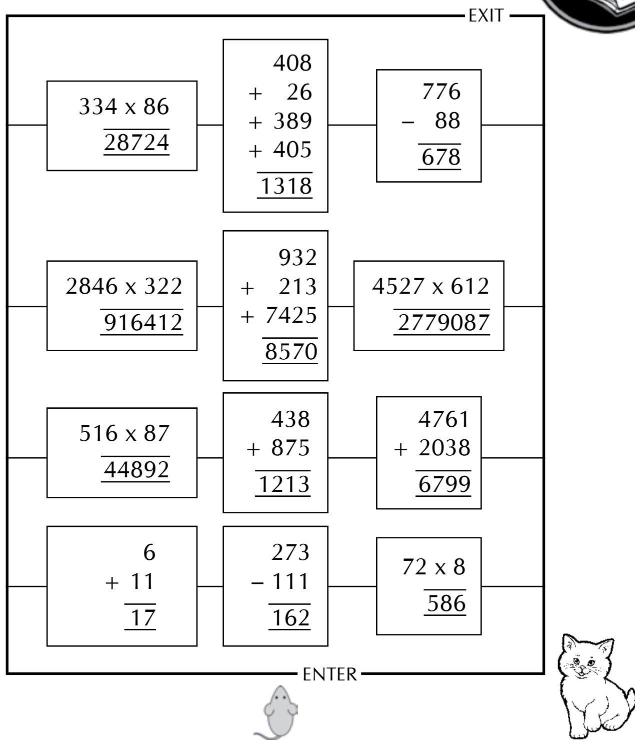
Here is a simple mathematical exercise based on the trick “casting out 9s”.

A cat has been chasing a mouse and they have just reached this den and the mouse is trying to escape by breaking the barriers in the den. It is quite easy, except that it needs to solve some mathematical problems to identify which gate is weak and can be broken.

In each layer, there are three problems, one of which has the wrong answer. That is the weak one. Your task is to use the method of “casting out 9s” to identify the incorrectly solved problem and break the gate open and help the mouse escape to the next layer. Do it four times and help the mouse go out through the exit gate at the top.

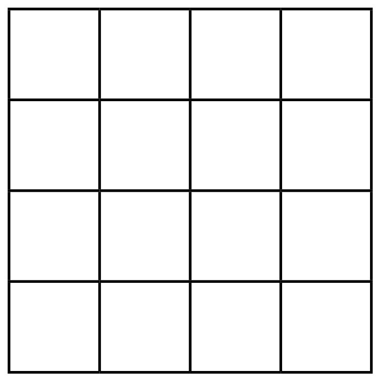
Can you create similar ones?

Casting out Nines



3. Single punch problem

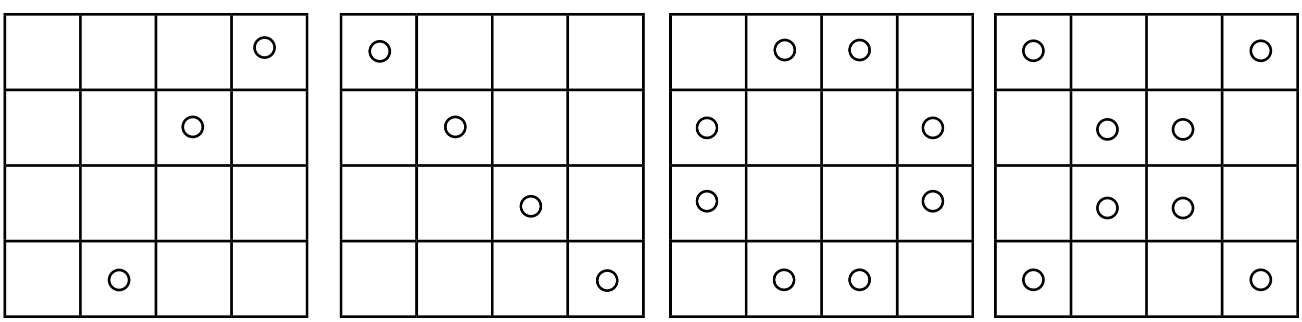
Mathematics is a curious subject where even with very little or no equipment or expensive and fancy materials, one can be deeply and meaningfully engaged for hours. This is a very interesting example using paper and single-hole punch.



Instructions are surprisingly simple.

1. Use only a square paper.
2. Fold the paper any number of times in any fashion; decision is yours.
3. Use a single-hole punch and punch just once.
4. Open up.
5. The result should match the corresponding picture in the worksheet.

You may want to extend this activity. The teacher must provide clear instructions on how to fold the square paper. The children are required to draw a picture of the resulting paper with holes in them before verifying the result.



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4. Happy numbers

Here is a simple iterative process that anyone with reasonable arithmetic skills can do easily and the results are surprising.

Activity:

1. Choose any natural number.
2. Square each digit of the number.
3. Add the squares to get a new number.
4. Repeat step 2 and 3 with the new number.
5. Find out if 1 is generated in this process. If yes, then the original number is a happy number. Else, it is a sad number.

E.g.

Let us choose 32

$32 \rightarrow 3^2+2^2 \rightarrow 9+4 \rightarrow 13 \rightarrow 1^2+3^2 \rightarrow 1+9 \rightarrow 10 \rightarrow 1^2+0^2 \rightarrow 1+0 \rightarrow 1$.

So, 32 is a happy number.

Let us choose 15

$15 \rightarrow 1^2+5^2 \rightarrow 1+25 \rightarrow 26 \rightarrow 2^2+6^2 \rightarrow 4+36 \rightarrow 40 \rightarrow 4^2+0^2 \rightarrow 16+0 \rightarrow 16 \rightarrow 1^2+6^2 \rightarrow 1+36 \rightarrow 37 \rightarrow 3^2+7^2 \rightarrow 9+49 \rightarrow 58 \rightarrow 5^2+8^2 \rightarrow 25+64 \rightarrow 89 \rightarrow 8^2+9^2 \rightarrow 64+81 \rightarrow 145 \rightarrow 1^2+4^2+5^2 \rightarrow 1+16+25 \rightarrow 42 \rightarrow 4^2+2^2 \rightarrow 16+4 \rightarrow 20 \rightarrow 2^2+0^2 \rightarrow 4+0 \rightarrow 4 \rightarrow 4^2 \rightarrow 16$

16 is recurring, so the process is interminable and number 1 can never be reached in this number chain.

So, 15 is a sad number.

• Check if the following numbers are happy or sad.

68 →

56 →

27 →

9 →

100 →

Further

1. Also, check if the following numbers are happy.
   17, 13, 19, 86, 7
2. Find all the happy numbers between 50 and 100.
3. Find out more about happy and sad numbers.

5. All digits – one out!

Take a good look at these sums that can pass for a typical page in a mathematics textbook. Beware, looks can be deceptive! Truth is, in these sums, **all (I repeat all)** the digits are wrong. Not even one digit is right. Your task is to correct each and every digit in these sums.

However, there is a method in the madness. Each wrong digit is one more or one less than the correct digit. This is the clue. Please go through the instructions carefully.

Hence

0 should be 1.

1 should be 0 or 2

2 should be 1 or 3

3 should be 2 or 4

4 should be 3 or 5

5 should be 4 or 6

6 should be 5 or 7

7 should be 6 or 8

8 should be 7 or 9

9 should be 8

Eg.
In the given question all the 6 digits i.e., 1,9,3,4,8 & 0 are wrong. Change all the digits.

$$\begin{array}{r} 19 \\ +\ \underline{34} \\ \underline{80} \end{array}$$

Here 8 should replace 9 & 1 should replace 0

$$\begin{array}{r} 18 \\ +\ \underline{34} \\ \underline{81} \end{array}$$

Only 3, not 5, should replace 4 because 8+5=13

$$\begin{array}{r} 18 \\ +\ \underline{33} \\ \underline{81} \end{array}$$

We arrive at the right answer when
2 replaces 1
4 replaces 3 &
7 replaces 8

$$\begin{array}{r} 28 \\ +\ \underline{43} \\ \underline{71} \end{array}$$

Correct the following expressions & find the right questions & answers

1. $\begin{array}{r} 50 \\ +\ \underline{61} \\ \underline{82} \end{array}$

2. $\begin{array}{r} 90 \\ +\ \underline{27} \\ \underline{86} \end{array}$

3. $\begin{array}{r} 210 \\ +\ \underline{119} \\ \underline{418} \end{array}$

4. $\begin{array}{r} 66 \\ +\ \underline{22} \\ \underline{99} \end{array}$

5. $\begin{array}{r} 3111 \\ +\ \underline{21010} \\ \underline{23232} \end{array}$

6. $\begin{array}{r} 43 \\ -\ \underline{34} \\ \underline{42} \end{array}$

7. $\begin{array}{r} 81 \\ -\ \underline{20} \\ \underline{92} \end{array}$

8. $\begin{array}{r} 94 \\ -\ \underline{30} \\ \underline{71} \end{array}$

9. $\begin{array}{r} 89 \\ -\ \underline{13} \\ \underline{47} \end{array}$

10. $\begin{array}{r} 333 \\ -\ \underline{222} \\ \underline{222} \end{array}$

11. 28 x 5 = 257

12. 36 x 4 = 250

13. 65 x 4 = 253

14. 72 x 3 = 361

15. 8 x 8 = 90

16. 43 x 20 = 461

17. 28 x 4 = 274

18. 4) 823 (353

19. 4) 716 (234

20. 2) 409 (215

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6. Simply simplify

The BODMAS rule is just a convention or a commonly agreed protocol to eliminate any confusion and uncertainties while simplifying a number expression with multiple operations like addition, subtraction, multiplication, division, exponents, etc. It is neither a mathematical concept nor a theory. You can also achieve the same results by suitably placing brackets (or parentheses). Here is one exercise based on this idea.

I. Check these for correctness
   a. 4 x (4 + 4) x (4 + 4) – (4 x 4) = 240
   b. 4 x ((4 + 4) x (4 + 4) – (4 x 4)) = 192
   c. (4 x 4) + (4 x 4) + (4 – 4)x 4 = 32

II. Place the parentheses (as many as you want) on the LHS (Left Hand Side) of the equation to get the number on the RHS (Right hand Side).
   a. 4 x 4 + 4 x 4 + 4 – 4 x 4 = 116
   b. 4 x 4 + 4 x 4 + 4 – 4 x 4 = 144
   c. 4 x 4 + 4 x 4 + 4 – 4 x 4 = 20
   d. 4 x 4 + 4 x 4 + 4 – 4 x 4 = -160

III. Evaluate. What number should appear on the RHS of the equations?
   a. 4 x (4 + 4) x ((4 + 4) – 4) x 4 = ___________
   b. ((4 x 4) + 4) x (4 + (4 – 4) x 4) = ___________
   c. 4 x 4 + 4 x 4 + 4 – 4 x 4 = ___________
   d. 4 x 4 + 4 x 4 + 4 – 4 x 4 = ___________

IV. Check these for correctness
   a. ((4 x 2) + 3) x 2 + (5–3) x 4 = 30
   b. ((4 x 2 + 3 x 2) + 5)–3 x 4 = 7
   c. (4 x (2 + 3)) x (2 + (5–3) x 4) = 200

V. Place parentheses (as many as you want) on the LHS of the equation to get the number on the RHS.
   a. 4 x 2 + 3 x 2 + 5 – 3 x 4 = 40
   b. 4 x 2 + 3 x 2 + 5 – 3 x 4 = 33
   c. 4 x 2 + 3 x 2 + 5 – 3 x 4 = 65

VI. Evaluate. What number should appear on the RHS of the equations?
   a. (4 x (2 + 3) x 2) + ((5 – 3) x 4) = ___________
   b. 4 x (2 + 3) x ((2 + 5) – 3) x 4 = ___________
   c. 4 x 2 + 3 x 2 + 5 – 3 x 4 = ___________
   d. 4 x 2 + 3 x 2 + 5 – 3 x 4 = ___________

7. Multi grid puzzle

This intriguing and attractive puzzle is based on the usual multiplication tables but the presentation is a bit different. The first row has the numbers from 1 to 12. So is the first column. However, they are not in that order and the numbers are placed randomly.

Your task is to place the numbers 1 to 12 in the first row and the first column so that the product of any two numbers appears in the cell that intersects the said row and column. Some cells are filled thus providing the necessary clue. After that, fill in the entire grid accordingly.

One example is made available.

2 x 9 = 18 →

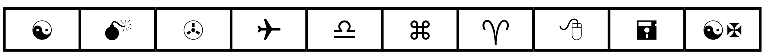
	8	4	3	9	7	5	11	2	1	6	10	12
4	32	16	12	36	28	20	44	8	4	24	40	48
11	88	44	33	99	77	55	121	22	11	66	110	132
1	8	4	3	9	7	5	11	2	1	6	10	12
2	16	8	6	18	14	10	22	4	2	12	20	24
5	40	20	15	45	35	25	55	10	5	30	50	60
6	48	24	18	54	42	30	66	12	6	36	60	72
8	48	32	24	72	56	40	88	16	8	48	80	96
9	72	36	27	81	63	45	99	18	9	54	90	108
7	56	28	21	63	49	35	77	14	7	42	70	84
10	80	40	30	90	70	50	110	20	10	60	100	120
3	24	12	9	27	21	15	33	6	3	18	30	36
12	96	48	36	108	84	60	132	24	12	72	120	144

	8											36
		10									70	
			8							24		
				56					42			
					10			11				
						25	15					
						40	24					
					120			132				
				24					18			
			44							132		
		9									63	
	12											54

	9											45
		36									12	
			48							66		
				14					42			
					120			70				
						110	44					
						10	4					
					24			14				
				24					72			
			24							33		
		72									24	
	5											25

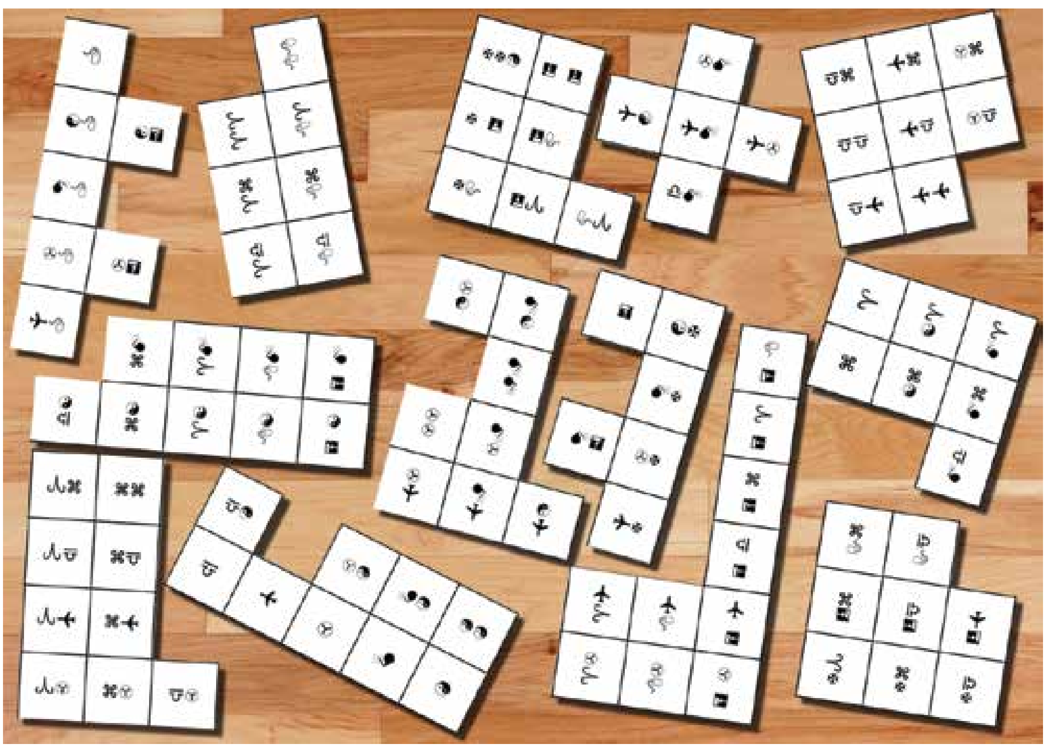
8. Cut it out!

Shambhavi had some homework to do. So she prepared a 10x10 grid with the numbers 1 to 100. However, in place of the digits 0,1,2,3 ....9 she used 10 different symbols and completed the grid. Then she went out to play. Meanwhile, her four-year-old brother came and started playing with the grid. He took out a pair of scissors and cut the grid into 13 pieces as shown in the picture.



When she came back, Shambhavi was furious but had to reconstruct the square grid to complete her homework. She started but found it really difficult.

Can you help her?



9. Palindromic numbers

Take a look at these words and sentences.

MALAYALAM	A Toyota. Race fast, safe Car. A Toyota.
CIVIC	Cigar? Toss it in a can. It is so tragic.
LEVEL	Ma is as selfless as I am.
ROTATOR	Never odd or even.
RACE CAR	No lemon, no melon.
RADAR	Step on no pets.
	A man, a plan a canal Panama.
	So many dynamos!
	Rise to vote, Sir.

Do you notice anything specifically interesting in these?

These words and sentences read the same whether you read from left to right or from right to left. This unique property has grabbed the attention of language enthusiasts over the centuries.

Students of mathematics form palindromic numbers with similar property. You may say "Oh! Big deal! Anyone can write palindromic numbers. It is child's play. I can write hundreds of them in no time". Yes, you are right. However, what we are going to do is a little different. We shall design a simple *mathematical process* to generate palindromic numbers from a seed number. Here it is.

Steps
1. Choose any TWO- digit seed number.
2. Check if it is a palindromic number. If no, then continue.
3. Reverse the digits and form a new TWO – digit number.
4. Add the original number and the one generated in step 3.
5. Check if it is a palindromic number. If no, then repeat steps 3 and 4 until you obtain a palindromic number.

Example

  32
+ 23
  55 Yes. 55 is a palindromic number.

Now, let us move over to three digit numbers.

   67
+ 76
  143
+ 341
  484 Yes. 484 is a Palindromic numbers.

It would be interesting to investigate this to understand and explore any possible pattern. Work out and fill up the following table.

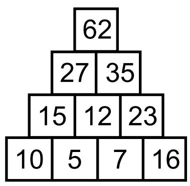
Sl. No.	Chosen number	How many iterations are needed?	The palindromic number thus generated
1	32	1	55
2	67	2	484
3	158	3	11011

4	48		
5	119		
6	167		
7	86		
8	184		
9	69		
10	169		
11	163		
12	52		
13	155		
14	78		
15	193		

Does every number eventually become a palindrome? Nobody knows for sure, since it has never been proven. For instance, 196 has been a very tough nut to crack, even today.

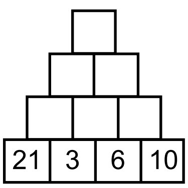
10. Pyramid math

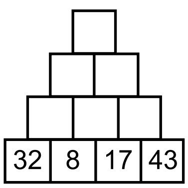
Did you ever notice that you have pyramids in history, pyramids in biology, pyramids in language and even in geometry but not in arithmetic? It is high time to introduce pyramids in arithmetic too. Here we go.

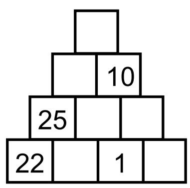


Notice the arrangement of the numbers here. The bottom row has four numbers. The numbers that are juxtaposed are added and the sum is written above it. Evidently, we get three new numbers in this process. The row above is generated similarly and at the end we are left with just one number at the vertex.

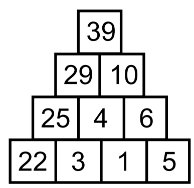
In the following number pyramids some numbers are missing. Can you find out what they must be?



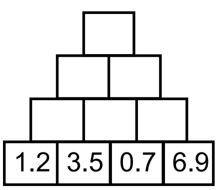


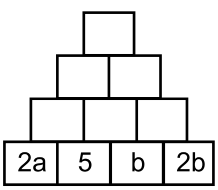


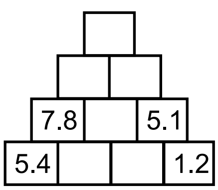
Here is the solution

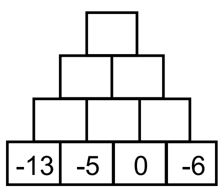


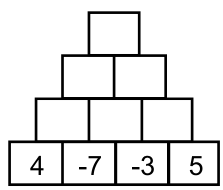
Also try these.

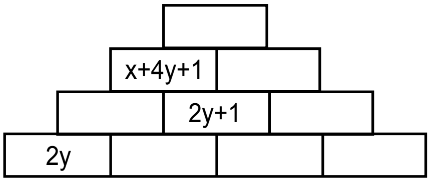


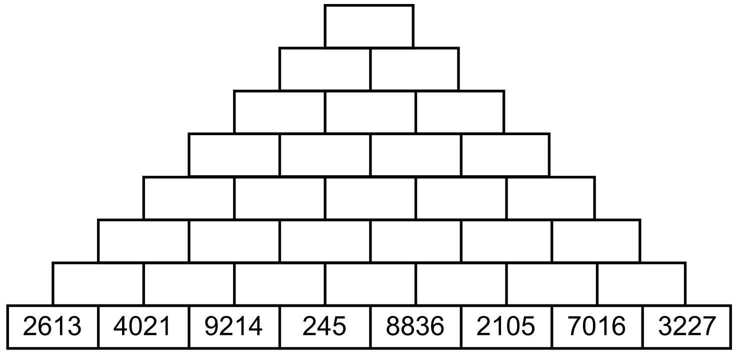












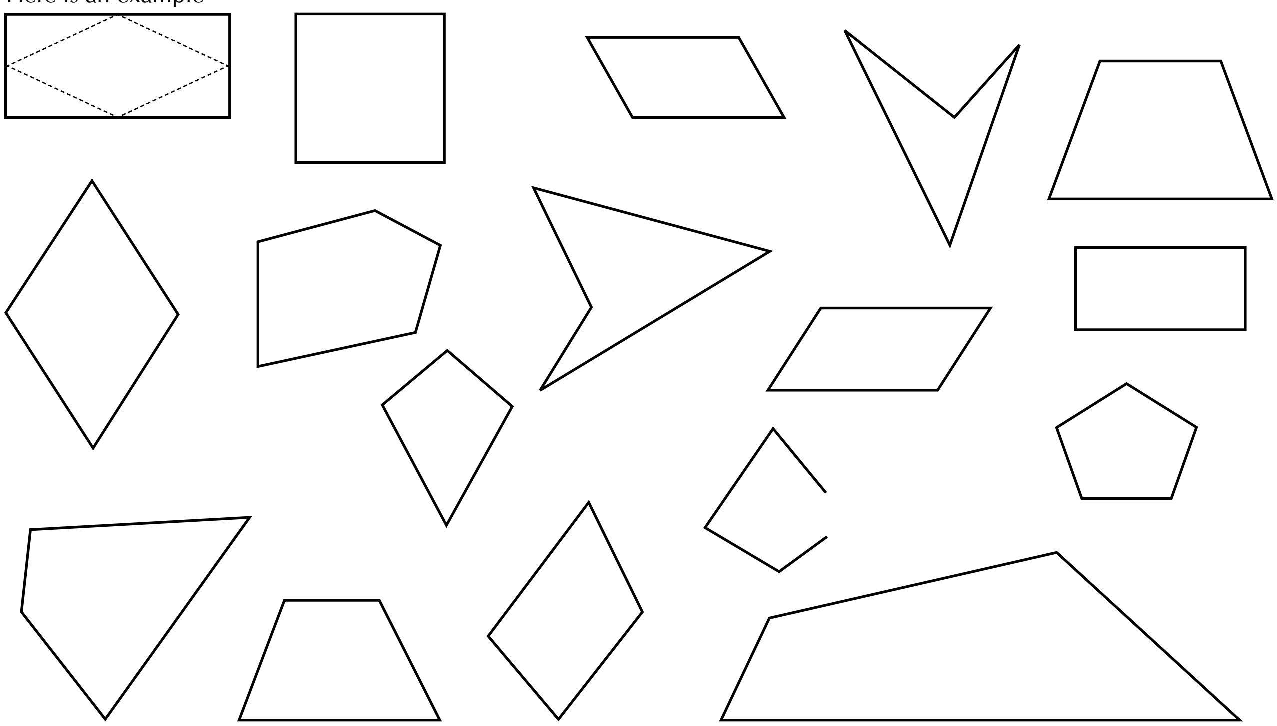
Was it fun? Can you create some on your own?

11. Mid-point mystery

Shall we explore a special property of quadrilaterals? (Note: Quadrilaterals are four sided closed shapes). Check and identify the quadrilaterals among these shapes and proceed.

- Mark the mid point of each side of the quadrilateral.
- Join the midpoints and identify the figure thus obtained
- Is it the same geometrical shape every time? Can you name it?
- Can you try this activity with triangles?

Here is an example



Do you get a parallelogram every time? What could be the reason for this?

12. Add up!

Here is a grid with many numbers in it. Is there a pattern in this arrangement of numbers? Think.

Here is your task.

1. Identify and encircle two, three or four numbers.
2. The chosen numbers must be contiguous (with no gaps between them).
3. They may be connected horizontally, vertically or diagonally.
4. Whatever numbers you choose must add up to a fixed number indicated at the centre of the grid.

Example. Here the expected sum is 150

I. Some 'answers' have been marked for you.
Search for more such numbers.

70	20	80	50	60	90	0	10
40	30	60	70	50	40	80	20
0	10	30	50	70	90	10	20
80	60	40	20	0	90	80	70
60	80	50	10	40	70	20	20
10	50	60	150		30	70	30
80	20	50			40	70	10
0	10	50	30	70	50	90	40
60	40	10	0	20	30	70	80
90	80	50	10	20	30	70	20
20	30	20	80	70	40	50	60
10	0	20	90	80	50	70	30

II. Total = 9

1	20	2	4	5	8	7	1
-10	-5	3	9	-1	3	2	7
-2	6	11	-3	11	6	6	5
-5	-1	13	5	20	-13	8	0
9	5	-2	7	8	-2	6	4
6	7	-2	9		10	-5	-1
15	8	4			8	9	5
1	-4	-2	15	3	6	-2	8
8	14	1	3	6	15	11	1
-4	7	-4	1	7	6	0	3
2	12	8	-2	6	4	8	2
1	4	9	10	-5	-1	6	7

Can you create similar problems suitable for different age-groups using for instance, integers, decimals, large numbers, and fractions etc?

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Teacher Plus Worksheet