Mathemagic for fun

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Activity 1: Estimation

(Suitable for students of all grades)

Estimation is an important skill in mathematics. We do it all the time unconsciously. For example, one estimates the distance of a vehicle and the speed at which it is moving from its shape and rate of change of its size, respectively. Similarly, while crossing the road, we know the right moment to cross by eliminating the possibility of a collision with a vehicle – that is as good as solving a linear equation in two variables. Here is an exercise to practice estimation and mental mathematics together, consciously.

Try to estimate the width of the river Ganga in this picture.

Photos courtesy: Vijay Anand

It is not easy to immediately guess the width, unless it is compared to a standard known length. From the picture, we can estimate that the width of the river Ganga is lesser than the width of the bridge on it. A good approximation can be made using the distance between each pillar supporting the structure. But we need the information on the distance between any two pillars.

After some time, a train crosses the bridge. This will helps us make a more accurate estimation. In the picture below, we can see the full length of the train on it. One can count the number of coaches on the width of the river. Let us say there are 23 coaches and one engine.

It is difficult to estimate the length of one coach, so we need to know some basic information about objects in the picture.

If one is given the following information, it becomes a little easier.

Description	ICF coaches (Integral Coach Factory)	LHB coaches (Linke Hofmann Busch)
Length over Body	21.77m	23.54m
Length over Buffers	22.28m	24.70m

So, taking the average length of a coach as approximately 23 meters. Multiplied by 24 (number of coaches here – length of the engine is approximately the same as that of a coach), we have 552 meters. Therefore, we can say that the approximate width of the river in this picture is about half a kilometer (550m). This is a rough estimate. In fact, if you want to be more accurate, one can refine the estimation by –

A. Using the length of the engine instead of assuming it to be the length of a coach.
B. Counting the exact number of coaches that fit in the width of the river, as it passes over the bridge.

Similarly, when one is sitting in a train, counting the number of power supply poles that whizz past and noting the time elapsed can help calculate the speed of the train. Of course, the distance between two poles needs to be known, which is approximately 100m. Have you ever tried that?

These kinds of exercises help in sharpening estimation skills and mental mathematics at the same time when you practice regularly. Objects that we use everyday can also be used to practice estimation in a fun way.

So, here are some more for you to exercise your mental mathematical skills.

Simple example

1. Estimate the length of the wire of the iron using the length of a standard foot ruler (*distance and angle of viewing of the two objects are same – so the perspective is preserved*).

2. Look at the picture below (Monsoon season picture)

It is known that the football field is of standard dimensions. Can you estimate the dimensions (length and breadth) of this sports field using the objects in the picture? Are there any differences in the perspectives of the sports field and the standard distance (between the football goal posts – 105m)?
{The perspective has to be same, otherwise the estimation errors are large}

What about estimation of the size of the same sports field from this picture? Is it possible? If yes, find the answer. If no, why not?

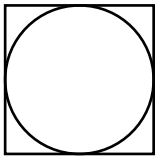
(Courtesy: Google Maps)

Can you find the dimensions of the solar farm (to the left of the sports field) using the same standard length? If yes, find the answer. If no, why not?
{*Here, the perspective is same, so the dimensions of the field can be estimated fairly accurately, but the use of a larger standard length – distance between football fields – to measure the dimensions of a smaller object will not give accurate results.*}

Activity 2: Estimating the value of Pi

(Suitable for students of grades 6 – 9)

Pi is a concept that has caught the imagination of people since ancient times. Its value is necessary to compute many important quantities about shapes with curvatures, such as understanding the relationship between a circle's radius and its circumference and area (circumference = $2\pi r$; area = πr^2). 3.14 or 22/7 that is known to us as the value of Pi is just an approximation. The number represented by Pi (π) is used in calculations whenever something round (or nearly so) is involved, such as for circles, spheres, cylinders, cones and ellipses.



There are some fun and simple methods for estimating the value of pi. One of the best-known methods is called the "Monte Carlo method". To try it at home, first draw a circle and then a square outside it, as shown, on a piece of paper. Make sure that the lines are not very thick.

Now pick up a pen, close your eyes and put dots on the square "at random" or drop a pencil with its lead pointing to the paper so that it leaves a mark on the object. Count the number of dots in the circle and the total number of dots (including those outside the circle but inside the square). If you make many dots by dropping the pencil enough times, and your efforts are truly random, the percentage of the number of dots inside the circle will approach 78.54% or 0.7854. Now, 4 x 0.7854 approaches the value of Pi (approx. 3.14).

Why is it so? What's the mathematics behind this?
{*Imagine the square's sides are of length 2 units, so its area is 4 square units; the circle's diameter is therefore 2 units, and its area is pi (radius is equal to one unit). The ratio between their areas is pi/4, or about 0.7854*}

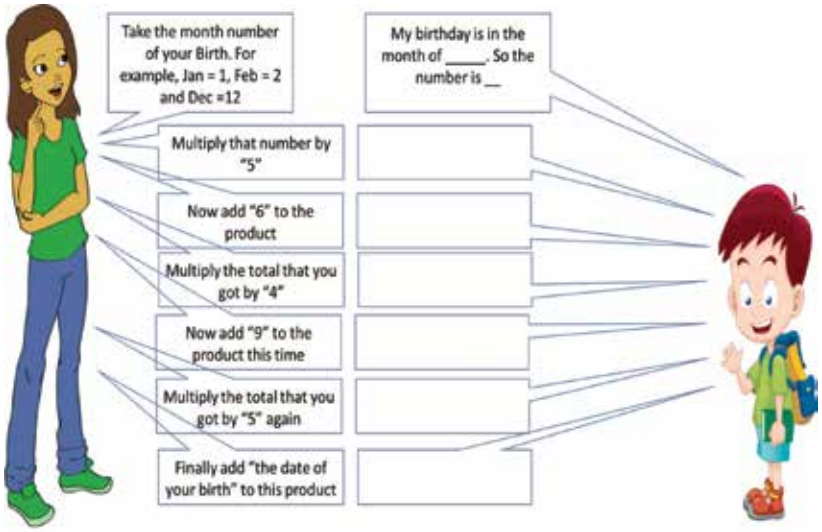
Activity 3: Guessing birthdays

(Suitable for students of all grades)

In the worksheet below, fill your answer based on the questions asked by the questioner in the box provided. At the end of it, the solution is given for teachers to first evaluate and then reveal the mathematics behind it to the students.

Try some examples with your elders and friends.

Solution: *Subtracting '165' from the final value obtained from the above, You get the month of birth in the place value of thousands and hundreds and date in the place value of tens and units.*



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Activity 4: Golden Ratio Activity

(Suitable for students of grades 5 – 8)

Many artists and designers use the Golden Ratio (phi, 1:1.618....) in their work simply because it is perceived to be the most aesthetically pleasing ratio that one can have. Many people go further and say that our bodies have been designed with phi integral to its design. Whether or not this is the case, phi is a very interesting ratio used in the real and natural world. So by exploring phi and its uses we are really doing 'Functional Mathematics' – using and applying mathematics in the real world. Credit cards, some ipods and photo frames and many more objects are designed to have a breadth to length ratio of 1:1.618 to appear pleasing to the eye.

Aim of the activity: The work considers some ratios within the body and determines whether or not they are approximate to the Golden Ratio (1:1.618....)

Activity plan: Before doing the activity, investigate what happens when two pairs of consecutive numbers are taken from the sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34....etc (Fibonacci numbers) and divide the larger number by the smaller number in each pair. By taking larger and larger numbers in the sequence, the answer to the division approximates to phi, 1.618....

Check it yourself:

1/1 = _____ 2/1 = _____ 3/2 = _____ 5/3 = _____ 8/5 = _____

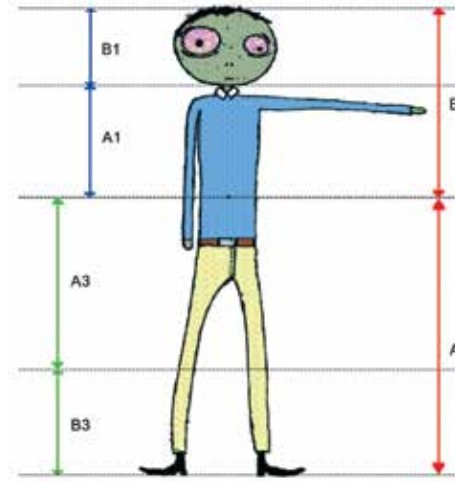
13/8 = _____ 21/13 = _____ 34/21 = _____ 55/34 = _____ 89/55 = _____

And so on.....

Resources and Materials: Measuring tape

How can this be adapted?

The Golden Ratio and the Fibonacci numbers are very rich topics to explore both in terms of algebra, geometry and links to the real and natural world.



Are you a Golden Person?

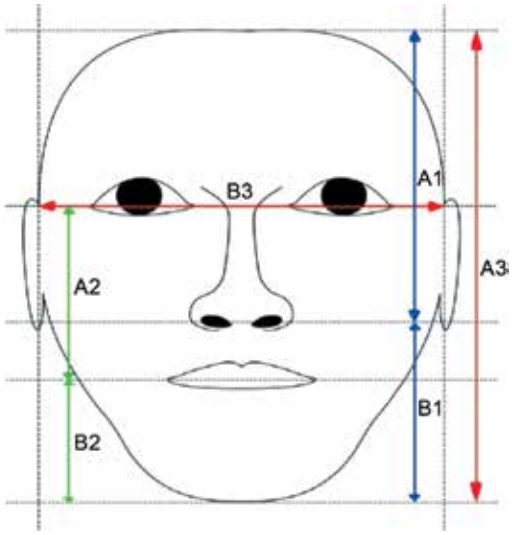
Look at the diagram below and then measure carefully the 3 sets of pairs on **your** body. Fill in the table below and work out the ratios/ divisions. You may take help from your brother or sister or parents or grandparents to hold the tape for you at one end.

Write the measurements in the space provided in the table

	Set 1 Data			Set 2 Data			Set 3 Data		
	Navel to neck	Head to neck	Ratio	Navel to feet	Head to navel	Ratio	Navel to knee	Knee to feet	Ratio
	A1	B1	A1/B1	A2	B2	A2/B2	A3	B3	A3/B3
Lengths measured in Cm									

What do you find? Are any of the ratios close to the value of Phi?

Now do the same exercise for your brother, sister, parents, grandparents, anybody in your family.



Do you have a divine or golden face?
Look at the diagram below and then measure carefully with a partner the 3 sets of pairs of your facial measurements. Fill in the table below and work out the ratios/divisions. You may take help from your brother or sister or parents or grandparents to hold the tape for you at one end.

Write the measurements in the space provided in the table

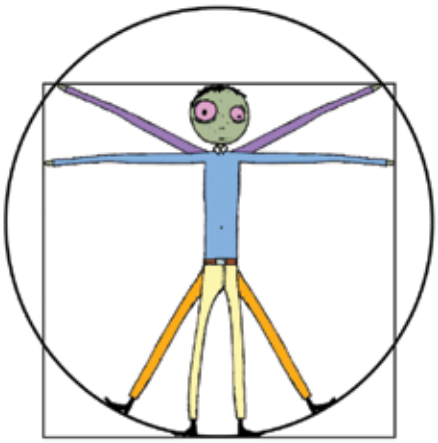
	Set 1 Data			Set 2 Data			Set 3 Data		
	Hairline to corner of the nose	Corner of the nose to Bottom of the chin	Ratio	Corner of the eye to corner of the mouth	Corner of the mouth to corner of the Chin	Ratio	Hairline to bottom of the chin (Height of the face)	One edge of the cheek to other edge (Width of the face)	Ratio
	A1	B1	A1/B1	A2	B2	A2/B2	A3	B3	A3/B3
Lengths measured in Cm									

What do you find? Are any of the ratios close to the value of Phi?

Now do the same exercise for your brother, sister, parents, grandparents, anybody in your family.

What about any famous people you like? Do they have 'divine or golden' faces? Use photos of two famous people to investigate.

Arm span and height investigation
Inspired by Da Vinci's painting *Vitruvian Man*, the picture below asks the question 'Is a person's arm span the same as his height?' Investigate by measuring your arm span and height, your parents', sisters' or brothers' (if any) and if possible of your grandparents.



Person	Arm Span	Height
Myself		
Mother		
Father		
Siblings (if any)		
Grandparents (if possible)		

What are your conclusions?

Activity 5: Area and perimeter of polygons

(Suitable for students of grades 6 – 8)

Materials required

a. Graph paper – one
b. Compass
c. Pencil
d. Scale

On the graph paper, draw the following, each separately

a. A circle
b. An equilateral triangle
c. A square
d. A regular hexagon

Measure the area and the perimeter of each geometric figure. Fill up the following table.

	Area	Perimeter	Ratio of Area to Perimeter (Area/Perimeter)
Circle			
Equilateral Triangle			
Square			
Regular Hexagon			

i) Which geometric figure has the highest ratio of area / perimeter?

ii) What does that mean? What conclusions can you draw from this information?

iii) Honeybees construct their hives in interlocked hexagons. What advantage do you think a hexagon offers over other geometric figures for building their hives? Will any other regular shape serve the purpose?

Activity 6: Animal algebra

(Suitable for students of grades 7 – 9)

The donkeys here are having a problem finding out the number of sacks they are carrying. Can you help them?

If you could do this one by trial and error, that's very good.

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Try the next one.

A Sheikh is counting the number of camels (with two humps each) and dromedaries (with one hump each) in a caravan of camels and dromedaries. He cannot see the bodies but is able to see the heads and humps and is counting them. Help him calculate the number of camels and dromedaries.

Activity 7: Craft activity

(Suitable for students of grades 7 – 9)

Aim – Verification of the algebraic identity I - $(a+b)^2 = a^2 + 2ab + b^2$

Materials required

a) Chart paper
b) 3/4 different coloured papers
c) Coloured sketch pens*
d) Pair of safety scissors
e) Ruler
f) Glue

Procedure: Take a coloured paper and draw a square of side '*a*' (you can choose the length) by measuring it. Take a second coloured paper and draw a square of side '*b*' (you can choose the length) on it. Take a third coloured paper and draw two rectangles of sides '*a*' and '*b*' on it. Now, using scissors cut the two rectangles and the two squares. Arrange them on the chart paper in the form of a big square of side $(a+b)$. Measure the area of the bigger square and compare it with the sum of the areas of the four figures.

Write the equation on the chart and the proof by substituting the value of '*a*' and '*b*' that you have used.
From what you have done, we can visually see the algebraic identity for $(a+b)^2$
Similarly, prove the algebraic identity $(a-b)^2 = a^2 - 2ab + b^2$

Write the equation on the chart and the proof by substituting the value of '*a*' and '*b*' that you have used.

Similarly prove the third identity $a^2-b^2 = (a+b)(a-b)$. Write the equation on the chart and the proof by substituting the value of 'a' and 'b' that you have used.

Prove the fourth identity $(x+a)(x+b) = x^2 + x(a+b) + ab$. Write the equation on the chart and the proof by substituting the value of '*a*' and '*b*' that you have used.

Activity 8: Poetry and mathematics

(Suitable for students of grades 6 – 10)

Fill in the blank by writing an equation for the poem below

A dozen, a gross and a score,

Plus three times the square root of four,

Divided by Seven,

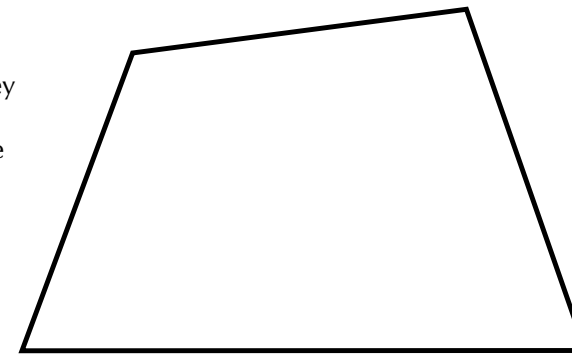
Plus five times eleven,

Is _____ squared and not a bit more

Activity 9: Problem of land division

(Suitable for students of grades 8 – 10)

Two brothers are part of an expedition and they have discovered a new island. From the sky they notice that the island is shaped like an **irregular convex quadrilateral**. They are not sure of the actual dimensions of the island, but they want to determine a way in which they can fairly divide the island between the two of them. (The figure here is just a representation)



The older brother suggests that they divide the land along one diagonal. Then, each can have one triangular part. He states that because this method creates two triangles and each will receive one of the two triangles, it is fair.

1. Do you think the two brothers should use this method? Explain why.

   _____

2. If the island was of a particular quadrilateral shape, would you suggest using this method? Explain in which shapes this method would work.

   _____

3. Determine another method for dividing the island fairly between the brothers. Explain why your method works. *{Hint: If you mark the mid points of each line segment in a quadrilateral and join them, you will always get a parallelogram and every parallelogram is made up of two identical triangles}*

   _____

4. The brothers want to make sure they each have a portion of their own, both on land and on water. Devise a method that will satisfy each brother. Explain your solution. *{Hint: Draw a diagonal and extend it beyond the island, into the sea, for a length equal to the diagonal. Connecting the nearest two vertices to the end of this extension of the diagonal gives four triangles. Sum of two triangles on the opposite sides of the diagonal will be equal to the sum of the other two. Use this method to find equal areas on land and sea}*

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This worksheet has been developed by Vijay Anand. He is a teacher and an administrator who likes Mathematics. He can be reached at <nv_andy@yahoo.com>. Additional information on his profile can be obtained from https://www.rajghatbesantschool.org/getinvolved/the-team/.